AP Calculus AB
Syllabus

Course Overview and Philosophy

The biggest idea in AP Calculus is the connections among the representations of the major concepts graphically, numerically, analytically, and verbally. The main focus of the course is the connections between these representations and the content. Although students are expected to use technology and memory to some extent, it is the "unifying themes of derivatives, integrals, limits, approximation, and applications and modeling" that make the course cohesive and not just a collection of random topics and memory tricks. Free-Response questions from previous exams are incorporated through-out the course with one or two problems given weekly. The free-response questions typically used are indicated under Evidence of Curricular Requirements as "FRQ", year, number. The use of free-response questions through out the year is one of the vehicles used to teach students how to express mathematical ideas in oral and written form. These problems are given with the understanding that students may work together and with the instructor to explore the focus of the problem and how various concepts of calculus are used to find solutions. As students discuss this in their groups in class and outside of class, ideas are explored that help students apply calculus. The written response to the question is graded using AP rubrics and returned to student to aid them in becoming more precise in their mathematical language.

The primary textbook used is Larson, *Calculus Early Transcendental Functions*, Fifth edition. The assignments included below each category come from this text unless otherwise indicated. Where indicated other resources are used and will be referred to by author. These texts include Finney, Demana, Waits, Kennedy, *Calculus, Graphical, Numerical, Algebraic*, 2003 and Hughes-Hallet, *Calculus Single Variable*. Any workbook pages mentioned in the assignments refer to a workbook made by the instructor which includes extra problems that rely on all the above resources and the experience of the teacher.

I. Functions, Graphs, and Limits

1. Review of Families of Functions: 7 Blocks (Blocks are 90 minutes in length)

The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions using a graphing calculator. Properties of functions will include:

a) Domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds; and
b) Intervals where the function is increasing or decreasing.

Assignments

Functions, Domain, Range	WB page 1, p.8 1,23,25-29,32

Domain, Range	WB page 2
Composite Functions	WB page 3
Trig Functions	p. A25 11, 15-18, 19, 20
Inverse Functions	p.43 1,3,5,6,9-12,42,44,91-98,107,121,123
Exponential Functions	p.52 11,13,15,47,49-54,61-75odd

Evidence of Curricular Requirements
Analysis of Graphs: With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function. Work here, especially with domain, range and sign analysis sets the stage for the investigation of limits and the continuity of functions. Students are asked to orally explain in class how the changes of signs influence the behavior of functions, especially rational functions. This explanation lays the foundation for the analysis of the first and second derivatives. Test problems require this analysis as well as a short written explanation that relates signs to function behavior.
FRQ 1982-2, 1991-4, 1995-1, 2000-6.

2. Limits: 3 Blocks
The student will define and apply the properties of limits of functions. This will include limits of a constant, sum, product, quotient, one-sided limits, limits at infinity, infinite limits, and nonexistent limits.

Assignments

Limits	p.72-73 1,5,7,11-18
Limits	WB p.8 Part I, p.83 13,19,32,
Limits	Page 84 69-81odd, WB page 8 II
One-Sided Limits	p.95 1-18, 25,26

Evidence of Curricular Requirements
Limits of Functions (including one-sided limits): Students will possess an intuitive understanding of the limiting process, be able to calculate limits using algebra, and estimate limits using graphs or tables. The use of sign analysis of factors in rational expressions with vertical asymptotes sets the stage for looking at signs of derivatives. Emphasis is put on the proper notation for taking limits. Discussion in class are followed by assignments in homework and free-response questions that emphasize the importance of limit notation and written expression of implications for the function based on limit findings. FRQ 1991-6, 1997-6, 1998-2

3. Continuity: 1 Block
The student will state the definition of continuity and determine where a function is continuous or discontinuous. This will include:

- Continuity at a point;
- Continuity over a closed interval;
- Application of the Intermediate Value Theorem; and
- Graphical interpretation of continuity and discontinuity.

Assignments

Continuity	p.95-96 37-51odd,63-66,83,91-93

Evidence of Curricular Requirements

Continuity as a Property of Functions: Students will possess an intuitive understanding of continuity, for example, close values of the domain lead to close values of the range. Students will understand continuity in terms of limits and possess a geometric understanding of graphs of continuous functions. FRQ 1973-3,1976-2,1978-3,1986-4,2002-6,2003-6

4. Infinite Limits/Limits at Infinity/Asymptotes: 2 Blocks

The student will investigate asymptotic and unbounded behavior in functions. This will include;

a) describing and understanding asymptotes in terms of graphical behavior and limits involving infinity; and

b) comparing relative magnitudes of functions and their rates of change

Assignments

Limits at Infinity	p.237 13-17
Infinite Limits	p.103 9-29 odd, 33-37,39-49 odd

Evidence of Curricular Requirements

Asymptotic and Unbounded Behavior: Students will understand the behavior of asymptotic functions and be able to contrast exponential growth, polynomial growth, and logarithmic growth. These behaviors will be explored using the graphing calculator and principles of sign analysis. Students are expected to look at one-sided limits and determine whether the function is increasing or decreasing based on factors and signs. This behavior is extended to finding slant asymptotes.
FRQ 1989-4, 1990-6, 1995-1

II. Derivatives

1. Definition of the Derivative: 1 Block

The student will investigate derivatives presented in graphic, numeric, and analytic contexts, and the relationship between continuity and differentiability. The derivative will be defined as the limit of the difference quotient and interpreted as an instantaneous rate of change.

Assignments

Derivative: Intro	p.120, 13-14,17-18,21,61,63,66,71-80

Evidence of Curricular Requirements
Concept of Derivative: The student should be able to relate to the notion of a rate of change using a graph, a table, or a function. Students should understand and articulate the difference between the average rate of change and the instantaneous rate of change. The connection with velocity is made here to illustrate the difference between the average and instantaneous values. Velocity is further discussed after the rules for finding derivatives are proven. FRQ1777-7, 1979-6, 1981-5, 1986-4, 2000-4, 2001-2

2. Derivative Rules: 8 Blocks
The student will apply formulas to find derivatives. This will include:

a) derivatives of algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions;
b) derivatives of sums, products, quotients, inverses, and composites (chain rule) of elementary functions;
c) derivatives of implicitly defined functions; and
d) higher order derivatives of algebraic, trigonometric, exponential and logarithmic functions.

Assignments

Rules	p.122 81,84,85 p.132 4-18even,25-29odd,48-52
Product/Quotient Rules	p.143 23,25,27,31,37,39,45,51,53/40-44,46-50
Other Derivatives	p.143-4 83-93 odd
Derivatives	p. 156 9-29odd
Chain Rule	p.157 59-77 odd, 81-89odd
Chain Rule	p.158 121-125odd,126,129-135odd
Chain Rule Notation	WB p.14
Implicit Differentiation	p.166 3,5,9,11,13,17
Inverse Functions	p.173 14,15,18-20,25-26

Evidence of Curricular Requirements
Computation of Derivatives: Students should be able to apply the fundamental algebra and properties of various functions to the simplification of derivatives. Additionally, students should understand the chain rule as the derivative of a composite function and extend this knowledge to using notation such as f(g(x)) and tabular data. FRQ 1973-3, 1978-2, 1978-5,1980-6,1990-2, 1990-6, 1992-4, 1993-3, 1995-3, 2000-5

3. Analysis of the Derivative: 6 Blocks

The student will analyze the derivative of a function as a function itself. This will include

a) comparing corresponding characteristics of the graphs of f, f ‘, f ‘’; (make the signs consistent)
b) defining the relationship between the increasing and decreasing behavior of f and the signs of f ‘;
c) translating verbal descriptions into equations involving derivatives and vice versa;
d) analyzing the geometric consequences of the Mean Value Theorem;
e) defining the relationship between concavity of f and the sign of f ‘; and
f) Identifying points of inflection as places where concavity changes and finding points of inflection.

Assignments

Extrema	p.203 21,24,25,26 p.219 13,19-29odd,34,39,40 p.227 9-13
Sketching	p.247 7,8,17,21,27,28,31,34,38 p.238 63,70
f, f ’, f ”	p.228 61-64, p.246 1-4
Rolle’s & Mean Value	p.210 10,11,14,17,21,27,29,37,38,40,42
	p. 312 and Finney, Demana, Waits, Kennedy, Calculus, Addison-Wesley Longman, 1999, p. 303-306

Evidence of Curricular Requirements

Derivative at a Function: Students should be able to interpret the sign of the first and second derivative and how it impacts the behavior of the function. Further, students should be able to relate the function and its first and second derivative verbally and graphically. Students are required to discuss f and f’ in terms of particular problems, not general recipe, with emphasis on the influence of the sign of f’ on the behavior of the function as well as how concavity relates to f’ increasing and decreasing. Students practice how to explain these relationships using graphs of f’ in exercises from their text as well as problems drawn from ancillary materials referenced earlier. The Mean Value Theorem and Rolle’s Theorem should be understood both geometrically and analytically. Rolle’s Theorem is presented first and then the Mean-Value Theorem. At this point students discuss in groups similarities between the theorems and they draw the conclusion that Rolle’s Theorem is a special case of the Mean-Value Theorem. FRQ 1979-7, 1985-4, 1989-1, 1993-1, 1996-1, 1997-4, 1999-4, 2000-3, 2001-4, 2002-6, 2003-4,2005-4

4. Application of the Derivative: 7 Blocks

The student will apply the derivative to solve problems

a) analysis of curves and the ideas of concavity and monotonicity;
b) optimization involving global and local extrema;
c) modeling rates of change and related rates;
d) use of implicit differentiation to find the derivative of an inverse function;

e) interpretation of the derivative as a rate of change in applied contexts including velocity, speed, and acceleration; and
f) geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Assignments

Velocity/Acceleration	p.135 91,92,94 Workbook page 19
Related Rates	WBp.17 6,8,9 p.180 19,23,24,26,27,30
Optimization	p.256 17,18,23,26,29,33,43WBp.31 1-4,7 WBp.32 5,6
Differentials, Linear Aprox.	p.267 8,9,15-17,33,35
Slopefields	Hughes-Hallett p.526- 531

Evidence of Curricular Requirements

Application of Derivatives. Students should be able to apply the derivative of various types to functions to various applied contexts. Slope fields provide an effective venue for students to discover how differential equations model functions. During this lesson, students are put into several groups and each is given a differential equation and a blank slope field graph. As a group they determine the slope field segments and then present their findings to the class. Students are then asked to interpret their slope field and describe the function it represents. The differential equations that can be solved at this point in the class are solved and students relate finding a particular solution to their slope field. FRQ 1985- 1, 2, 5, 1986-1, 2, 3, 1987-1, 2, 3, 4, 5, 6, 1988-1, 2, 4, 1989-1, 3, 4, 5, 1990-1, 4, 5, 1991-3, 5, 6, 1992-1, 2, 3, 6, 1993-1, 2, 4, 5, 1994-1, 4, 1995-2, 5, 6, 1996-1,4,5,6, 1997-1,2,4,5,2002-5,2003-2,2003-5,2005-2,2005-6

III. Integrals

1. Indefinite Integrals: 3 Blocks

The students will find antiderivatives directly from derivatives of basic functions and by substitution of variables (including change of limits for definite integrals).

Assignments

Basic Integration	p.283 9-43 odd
Differential Equations	p.284 53,54,59-69

Evidence of Curricular Requirements

Techniques of Antidifferentiation: Students should be able to evaluate definite and indefinite integrals with and without a calculator. Students should understand the significance of the boundaries of integration. These techniques are then applied to solving differential equations. Graphical interpretations of dy and delta y are explored graphically with applications of local linearization and use of the tangent line for estimation. Student homework focuses on finding dy and delta y and using written descriptions to explain the difference between exact change and approximate change. Student discussion in class leads the group to make connections between this topic and local linearization. FRQ 1972-7, 1991-1

<u>2. Riemann Sums and Area:</u> 3 Blocks

The student will use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, graphically, and by a table of values, and will interpret the definite integral as the accumulated rate of change of a quantity over an interval interpreted as the change of the quantity over the interval a to b the integral f(x)dx = f(b) – f(a). Reimann sums will use left, right, and midpoint evaluation points over equal subdivisions.

Assignments

Riemann Sums	WB p.39
Trap.Rule/Def Integral	p.340 4,5,10 p.318 5-21odd
Riemann Sums again	p.318 23,25,p.306 23-31 odd, 45 Kennedy p.254 9-13

Evidence of Curricular Requirements

Interpretation of Integrals: Students should be able to use Riemann and Trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by table of values and give written explanation of their meanings**.** Extra time is taken with these topics to fully explore the notion of sums representing approximations of area. Students use calculator programs or Calculus Tools on the TI89 to find and illustrate left, right and midpoint sums. Additionally, area is approached using functions, data from tables, and graphically using formulas from geometry. Because this topic is introduced conceptually as a limit of sums, students are able to articulate the difference between Riemann approximations and the definite integral. This difference is emphasized in the assignments that require students to evaluate definite integrals geometrically versus the use of tabular data to approximate area. Through discussion in class, students come to the realization that written explanation of the definite integral must address that it represents the total accumulation in units over a specific interval. The students practice this written explanation in homework and on free-response questions that support this topic. **FRQ** 1998-3, 1998-5,1999-3,2000-2,2001-2, 2003-3, 2003-4,2005-5

<u>3. Fundamental Theorem of Calculus</u>: 3 Blocks

The student will identify the properties of the definite integral. This will include additively and linearity, the definite integral as area, and the definite integral as a limit of a Riemann sum as well as the fundamental theorem:

$\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$ Additionally, the student will use the Fundamental Theorem of Calculus to evaluate definite integrals and to represent a particular antiderivative as well as the analytical and graphical analysis of functions so defined

Assignments

FT of C	Kennedy, p. 255-6 19,22,24,26

FT of C & Average Value	p.320 93-98, 99, 101 p.318 51,53

Evidence of Curricular Requirements
Fundamental Theorem of Calculus: Students will use the Fundamental Theorem of Calculus to evaluate definite integrals and analytical and graphical analysis of functions defined by a definite integral with a variable upper bound. Applications will continue to include velocity and acceleration with accumulating functions. Problems taken from the Kennedy text reference earlier as especially useful in making the connection between the accumulating function and the graph of the derivative. These problems along with free-repsonse from the last few years give students another opportunity to make connections between f, f' and f '. Students give written explanations that relate the sign of f ' to the behavior of f and the behavior of f ' to the concavity of the function using graphs of the derivatives of functions with a variable upper bound. Average value is also demonstrated graphically with students making connections with area and the conclusion of the average value theorem. FRQ 1995-6, 2001-3, 2002-4,2003-4,2005-4, 1996-5, 1997-5, 1999-5, 2002-2

4. Applications of the Integral: Differential Equations : 4 Blocks
The student will find specific antiderivatives using initial conditions (including applications to motion along a line). Separable differential equations will be solved and used in modeling (in particular, the equation y ' = ky and exponential growth.)

Assignments

Growth & Decay/Acceleration	WB p. 50-51 and page 35-6

Evidence of Curricular Requirements
Applications of Antidifferentiation: Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Emphasis on reading these problems includes students identifying information relating to growth/decay constants and adapting the general differential equation to the problem. A good example always included is the wolf population problem from a prior AP exam. Student discussion in class gives them an opportunity to discover Newton's Law of Cooling and they are able to explain how ambient temperature influences these problems. FRQ 1996-3, 1997-6, 1998-4, 1999-4, 2000-6, 2003-5

5. Applications of the Integral: Area and Volume: 5 Blocks
The student will use integration techniques and appropriate integrals to model physical, biological, and economic situations. The emphasis will be on using the integral of a rate of change to give accumulated change or on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications will include

a) the area of a region;

b) the volume of a solid with known cross-section;
c) the average value of a function; and
d) the distance traveled by a particle along a line.

Assignments

Area	p.419 17,19,21,22,32
Volume: Disc	p.428 11 a,c 12 b,c 22,27 p.419 33,34,46
Volume: Washer/ Known Cross Sections	p.428 11 b,d 12 a,d 13,16,20,24 p.431 60

Evidence of Curricular Requirements

Applications of Antidifferentiation: Models are used with students to help them discover how to extend area to volume by revolving solids. Students are put in groups with manipulatives representing the cross sections (disc or washer) and discuss what the integral must look like to give volume. Students discover and explain orally the use of volume of a disc in terms of the model and relate the radius of the solid to the length of the rectangle used in area applications. FRQ 1998-5, 1999-3, 2000-1, 2000-4,2001-1, 2002-1, 2002-3, 2003-1, 2003-2,2005-5

IV. Review for the AP Exam 8 Blocks

Students are given released questions from past AP exams and 4 free-response questions prior to spring break. Students may work in groups or with the teacher to clarify questions and their answers, but must answer questions on their own during the review in class. When students return from spring break, the "AP Extravaganza" begins. 25 questions are due each day and students are called upon to go in front of the class to explain the reasoning behind the answer to the question or questions they must answer. Students realize that the teacher will ask subsequent questions they must answer to insure that the proper connections are established that support their reasoning. Grading students on accuracy, clarity of explanation, and making proper connections between the various concepts involved in the question raises the level of concern and each year students work very diligently to perform well in front of their peers and on the AP exam. Free-response questions are done weekly and graded, so the final questions given in this packet are chosen carefully to require students to continue to write appropriate responses. Written responses are solicited through out the year through AP questions and open-ended questions on tests; consequently, these last questions chosen always include previous AP questions that require written analysis.

V. Topics Covered After the AP Exam 8 Blocks

1.Techniques of Integration are continued. Students learn how to use the following techniques:

a. Integration by Parts and Repeated Integration of Parts
b. Trigonometric Substitution
c. Partial Fractions
d. Miscellaneous Substitution

Assignments

Techniques of Integration	p.494 5,12,13,15,16,18,24,25,27,34,35,49
Techniques of Integration	p. 512 7,12,19,25,26-32 even 37,41
Techniques of Integration	p.512 28,30,37,41
Techniques of Integration	p.522 7-15odd 18,21

Evidence of Curricular Requirements

Students are taught to analyze each integral for properties that lead them to identify the technique of integration used based on how the integral is constructed. As each technique is taught, students are asked to determine why previously learned techniques fail and to identify what critical attribute the integral possesses that helps identify the technique to apply.